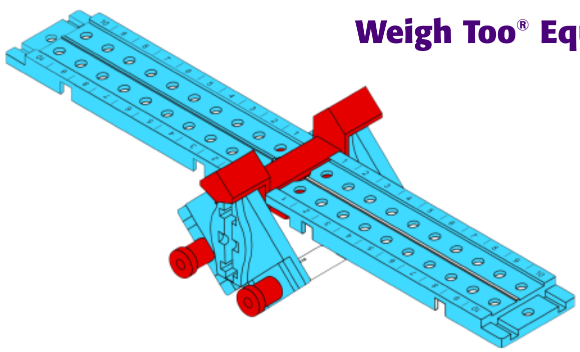
Introduction

The study of algebra must do more than focus on manipulating symbols across equations. Such a dance of the symbols can be too abstract to enable students to fully comprehend the key relationships and procedures. As a result, students have difficulty building foundational understandings of equations and methods for solving them. In ***Principles and Standards for School Mathematics,*** the National Council of Teachers of Mathematics (NCTM) suggests using manipulatives to assist students to build solid understanding of mathematics concepts.

"Students generate solutions that are based on number sense and properties of operations that use a variety of models and representations."—NCTM

In the case of equations, NCTM suggests that a balance and cubes can aid students to understand the equality aspect of solving equations. The Weigh Too® Equation Balance™ provides students with such an opportunity. It allows students to fully explore the concrete balance aspect of equations. When students employ the balance to solve for the variable in an equation, they strengthen their understanding of an equation as an equality or balance between two quantities.

Weigh Too® Equation Balance™



This text contains a variety of algebraic concepts that utilize the balance, ranging from simple single-step equations, to solving some forms of quadratics. Students can use the balance to represent algebraic relationships, solve problems, and check answers to equations. Thus, the balance may be used to introduce a concept, explore a concept, demonstrate a relationship, or serve as a useful means to review or remediate algebraic understandings.

SECTIONS:

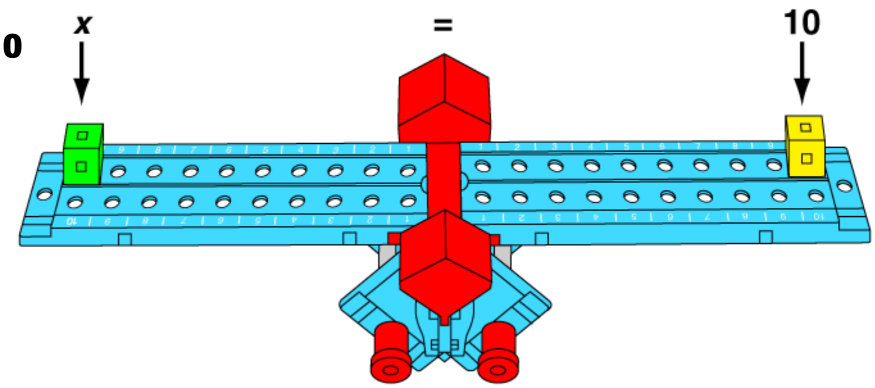
Step 1: Model

Balancing Equations

When modeling an equation during the simplifying and solving process, the top portion of the Equation Balance should be removed to lay flat on the desktop for easy addition and removal of cubes. Return the top portion to the Equation Balance base to verify the solution and the original equation.

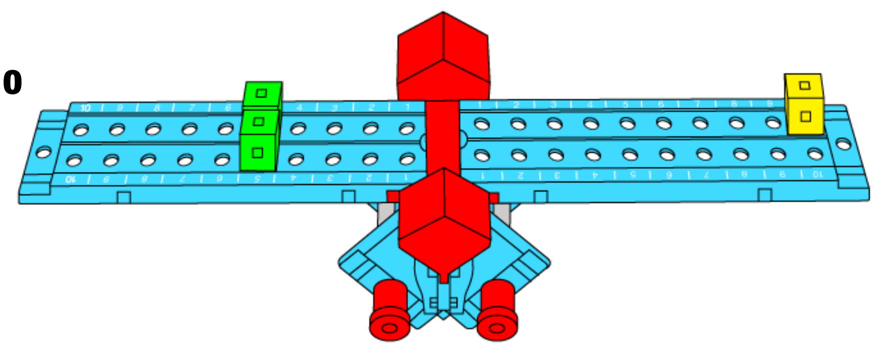
The yellow cube is placed in the 10 position on the right side of the balance and is “10”. The red fulcrum represents the equation symbol for “=”. To make the equation balance, the “*x*” cube (dark green) must be put into the 10 position on the left side of the equation balance. The balance below models $x = 10$.

$x = 10$



The yellow cube is in the 10 position on the right side and two dark green cubes placed in the 5 position on the left side will make the equation balance. The balance below models $2x = 10$.

$2x = 10$



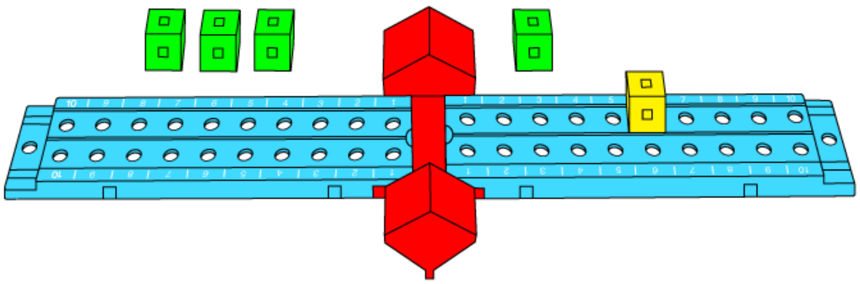
Writing Equations

It is important that students learn to write the equations and operations while they perform the modeling on the Weigh Too® Equation Balance™. Students may use the balance to first represent the equation, and then employ the balance to solve it as they write the equations. Other students might write the equations to solve the problem first, and then use the balance to verify their answers. Either method will show the students' understanding of number concepts both physically and symbolically.

Step 2: Simplify & Solve

What if our equation has an x on both sides, as in $3x = x + 6$? Begin by placing the number lines (top portion of equation balance) on the tabletop. Since we don't readily know the value of x, or what position to put our x cubes in, model the expression by placing three dark green cubes above the left side of the equation balance (not fixed into any number position). Place one dark green cube above the right side of the equation balance, and one yellow cube in the 6 position. This models $3x = x + 6$.

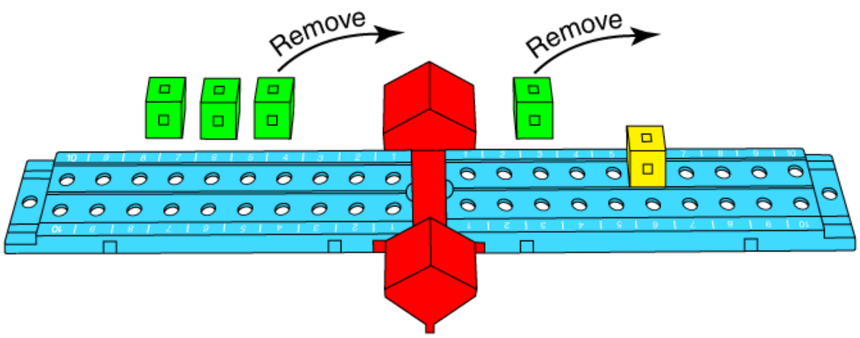
$3x = x + 6$



$3x = x + 6$

Remember that both sides of the equation must be of equal value (The Balance Principle) and that whatever we add or subtract from one side, we must also do to the other side. Because there is at least one x on both sides, we can remove one x (dark green cube) from both the left and right side of the balance scale. We are now left with $2x = 6$.

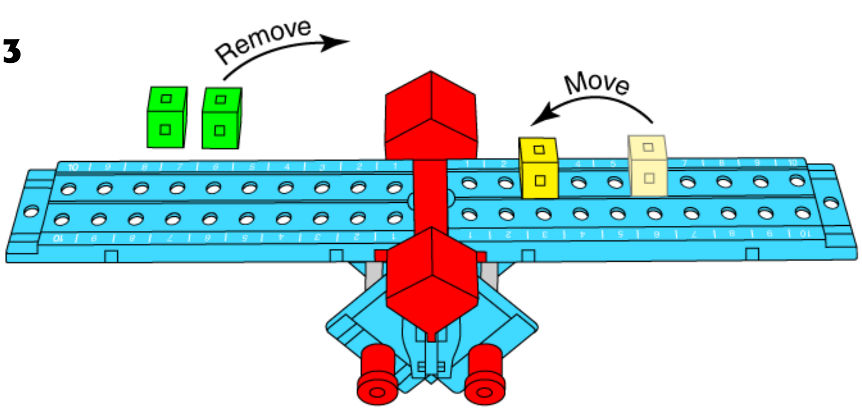
$2x = 6$



$$\begin{aligned} 3x &= x + 6 \\ -x &= -x \\ \hline 2x &= \quad 6 \end{aligned}$$

At this point, think about how taking cubes off each side of the balance is like 'undoing' the equation. For example, in the equation $2x = 6$, you need to 'undo' the 2 in the $2x$ to find the value of x alone. You know that $2x = 6$, to find the value of x alone you need to find the value of x before it was multiplied by 2. One way to do this is to think of dividing by 2 as a way of 'undoing' the x in $2x$. According to the Balance Principle, that would mean dividing both sides of the equation by 2 to maintain the balance. On the equation balance this operation would be modeled by removing an x cube from the left side and changing the yellow integer cube from the 6 position to the 3 position. Move the top portion of the Equation Balance from the desktop to the balance base. Notice that the x on the left when placed in the 3 position physically balances the yellow 3 on the right. This would leave a model of $x = 3$.

$x = 3$



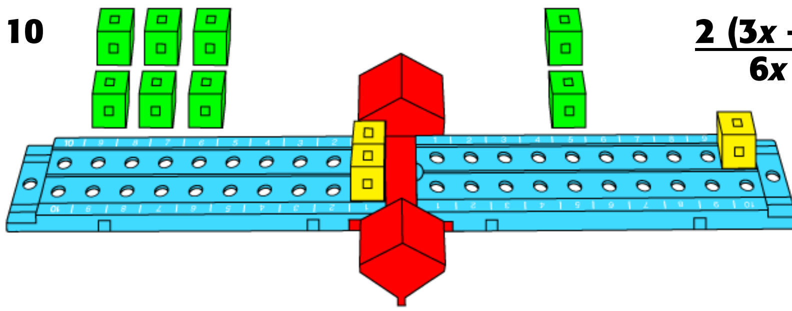
$$\begin{aligned} 3x &= x + 6 \\ -x &= -x \\ \hline 2x &= \quad 6 \\ 42 &= \quad 42 \\ \hline x &= \quad 3 \end{aligned}$$

NOTE: The written solution of $3x = x + 6$ is located to the right of the model.

Another Example: $2(3x + 1) = 2x + 10$

1. Since we don't readily know the value of x, or what position to put our x cubes in, model the expression by placing, on the left side, three dark green cubes above the equation balance and one yellow cube in the 1 position. We do what is in parenthesis first.
2. Repeat step one because the equation requires you do what is in the parenthesis two times. This means you will need 6 dark green cubes above the left side of the balance with two yellow cubes in the 1 position.
3. Place two dark green cubes above the right side of the equation balance and one yellow cube in the 10 position. This models $2(3x + 1) = 2x + 10$.

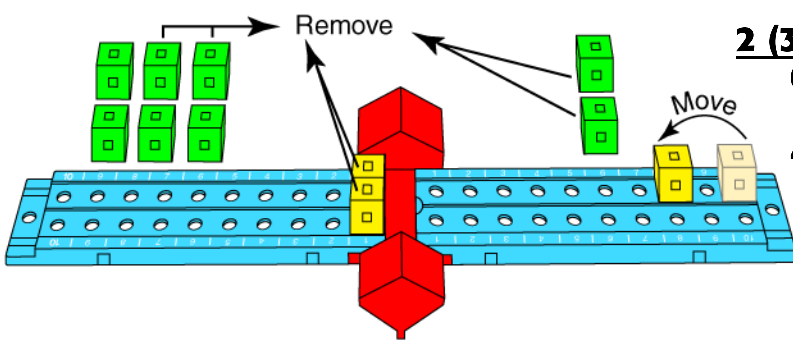
$6x + 2 = 2x + 10$



$$\underline{2\,(3x + 1) = 2x + 10}$$
$$6x + 2 = 2x + 10$$

4. Remember that both sides of the equation must be of equal value (The Balance Principle) and that whatever we add or subtract from one side we must also do to the other side. Because there are at least two x cubes on both sides, we can remove two x cubes (dark green cubes) from both the left and right side of the balance scale. We are now left with $4x + 2 = 10$.
5. Our goal is to isolate the variable. Therefore, we should take away 2 from both sides of the balance. This leaves $4x$ on the left and 8 on the right.

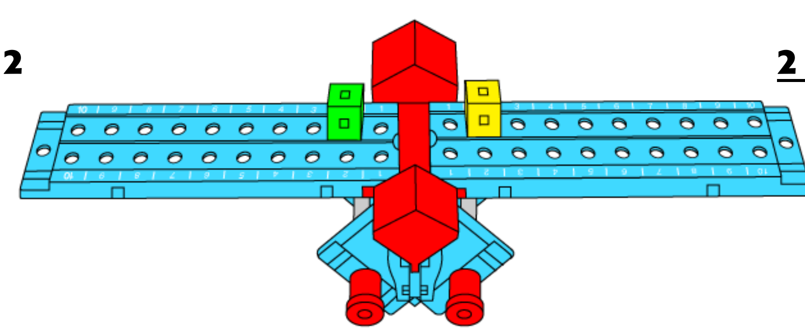
$4x = 8$



$$\underline{2\,(3x + 1) = 2x + 10}$$
$$6x + 2 = 2x + 10$$
$$\underline{-2x = -2x}$$
$$4x + 2 = 10$$
$$\underline{-2 = -2}$$
$$4x = 8$$

6. To find the value of x alone you need to find the value of x before it was multiplied by 4. One way to do this is to think of dividing by 4 as a way of 'undoing' the x in $4x$. According to the Balance Principle, that would mean dividing both sides of the equation by 4 to maintain the balance. On the Equation Balance this operation would be modeled by removing 3 "x" cubes from the left side and changing the yellow integer cube from the 8 position to the 2 position. This would leave a model of $x = 2$.

$x = 2$

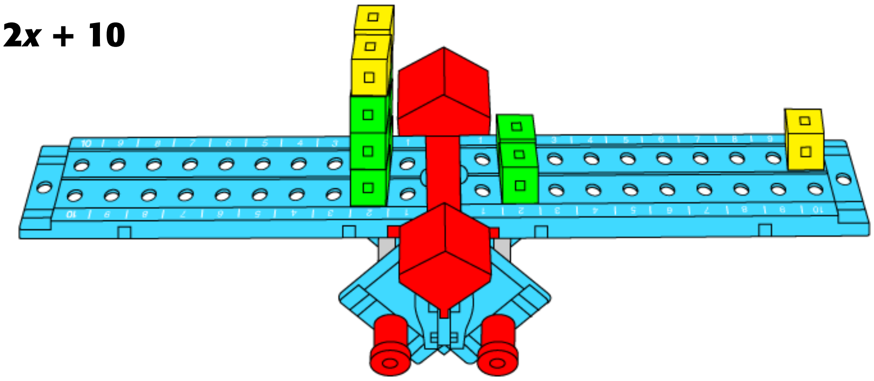


$$\underline{2\,(3x + 1) = 2x + 10}$$
$$6x + 2 = 2x + 10$$
$$\underline{-2x = -2x}$$
$$4x + 2 = 10$$
$$\underline{-2 = -2}$$
$$\underline{4x = 8}$$
$$\underline{4\,4 = 4\,4}$$
$$x = 2$$

Step 3: Check

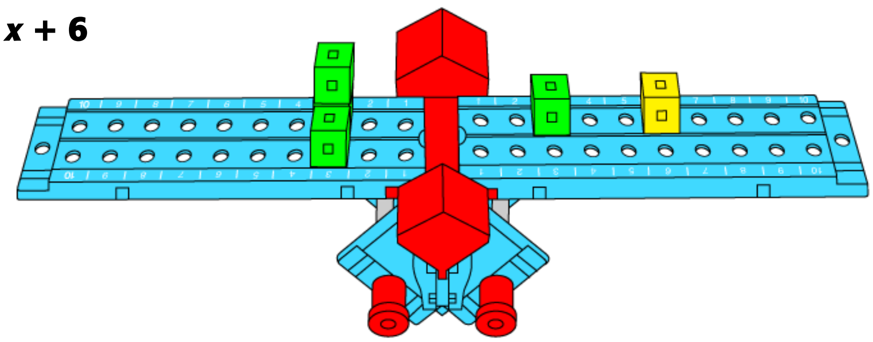
To further aid in understanding the physical aspect of number relationships, students should model the original equation. Below is a model of the problem from page 8.

$2(3x + 1) = 2x + 10$ **$x = 2$**



Below is a model of the problem from page 7.

$3x = x + 6$ **$x = 3$**



Solving Equations

Represent and solve these equations using the balance.

1. $y + 5 = 8$	**2.** $x + 8 = 14$	**3.** $y + 2 = 5$	**4.** $x + 4 = 8$
5. $y + 6 = 15$	**6.** $15 = q + 6$	**7.** $11 = z + 5$	**8.** $t + 5 = 12$
9. $r + 5 = 14$	**10.** $y + 6 = 18$	**11.** $13 = r + 6$	**12.** $q + 5 = 16$
13. $x + 3 = 7$	**14.** $13 = 2 + x$	**15.** $y + 4 = 7$	**16.** $x + 6 = 9$
17. $y + 3 = 15$	**18.** $x + 3 = 8$	**19.** $y + 4 = 10$	**20.** $15 = q + 8$
21. $12 = z + 6$	**22.** $t + 6 = 11$	**23.** $r + 6 = 14$	**24.** $y + 8 = 10$
25. $14 = r + 8$	**26.** $q + 5 = 10$	**27.** $12 = q + 4$	**28.** $x + 7 = 12$

Challenge A: $y + 5 + 3 = 12$

Challenge B: $y + 6 + 7 = 18$

Solutions: Solving Equations

1. 3	**2.** 6	**3.** 3	**4.** 4
5. 9	**6.** 9	**7.** 6	**8.** 7
9. 9	**10.** 12	**11.** 7	**12.** 11
13. 4	**14.** 11	**15.** 3	**16.** 3
17. 12	**18.** 5	**19.** 6	**20.** 7
21. 6	**22.** 5	**23.** 8	**24.** 2
25. 6	**26.** 5	**27.** 8	**28.** 5

Challenge A: 4 **Challenge B:** 5