



Pre-Algebra & Algebra I
Using
Algebra Models™

Table of Contents

Preface	1
Unit One	
Introduction and Exploration	4
Identifying and Sketching the Tiles	5
Building Rectangles and Squares	10
Representing Zero	12
Addition and Subtraction	15
Multiplication	24
Division	30
Unit One Assessment	35
Unit One Selected Answers	36
Unit Two	
Solving Equations	38
Working with Fractions and Proportions	46
Properties	52
Linear Equations	57
Unit Two Assessment	62
Unit Two Selected Answers	63
Unit Three	
Multiplying Polynomials	65
Dividing Polynomials	74
Factoring Polynomials	81
Perimeter	87
Area and Perimeter	89
Unit Three Assessment	91
Unit Three Selected Answers	92
Unit Four	
Review and Practice	94
Unit Four Answer Key	117
Mats	123

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Send all inquiries to:
Classroom Products™
P.O. Box 26
Bloomingdale, IL 60108

Editor: Judy Mendoza
Art Director: Gary Hanke
Design and Layout: Carol Davis

Preface

Introduction

Algebra Models™ is a complete manipulative program for teaching algebraic concepts. A classroom set of Algebra Models™ includes 15 student sets of 110 tiles, an overhead set, and a 128-page teacher's activity guide including all necessary blackline masters. This unique set of manipulatives consists of 45 blue unit tiles, five blue 5-unit tiles, ten aqua x^2 tiles, five green xy tiles, fifteen purple y tiles, twenty-five aqua x tiles, and five purple y^2 tiles.

These exciting manipulatives are designed to model algebraic concepts through an area model making evident the connection between algebra and geometry. Algebra Models™ builds students' confidence, motivates them to explore more complex mathematical problems, and inspires them to communicate, both orally and in writing, about mathematical situations.

Algebra remains the gateway into higher-level mathematics classes for students; yet mastery of high school mathematics continues to be elusive to many students. Algebra is difficult and often too abstract for many students. Using the Algebra Models™ program is an effective method to build understanding of complex concepts. Algebra Models™ offers a concrete and informal approach to key algebraic concepts, giving students an additional way to explore and become successful with algebra. Success builds confidence! Confidence builds strong risk-taking students who look forward to the challenges that higher-level mathematics courses offer.

In order to be successful in any pre-algebra or algebra I class, students should have a working knowledge of basic number systems, principles and properties of mathematics such as integers, commutative and associative properties, and the zero principle. Using Algebra Models™ will not change the need to have a working knowledge of these concepts, therefore some background work may be necessary prior to beginning any algebra course. Many students tend to forget these skills and concepts. A review is recommended as necessary throughout the school year.

Algebra is an extension of arithmetic. Algebra Models™ is designed to incorporate a proven method used

when teaching mathematics in successful elementary classrooms. Students will build, sketch, write, and explain their thinking. Successful students own all areas of this process. The first step is building and finding solutions to an equation using manipulatives. Successful students sketch what they have built, write the number sentence, expression, or equation, and finally explain their thinking both orally and in writing throughout the process.

Example: To solve the problem $x - 3 = 5$, students model the equation using Algebra Models™, sketch what they have modeled, write an expression, equation, or number sentence, and explain their thinking throughout the process. If students are to develop an understanding of algebraic concepts they should be building, sketching, writing, and explaining their thinking each step of the way. Making the connection between the manipulative (concrete) and the abstract (symbolic notation) is critical to this process.

Algebra Models™ offers a springboard to communication and exploration; it is as important for students to explore why something doesn't work, as it is to explore why things do work.

Limitations

As with any physical model, Algebra Models™ will not do everything or every problem within a given set of problems. However, Algebra Models™ will offer you and your students a springboard for communication and exploration into more complex mathematical ideas. Algebra Models™ will aid in developing a visual model for complex situations. For example, as students are attempting to combine like terms such as $x - 2x^2 - y - 3x - x^2 - 4y$, Algebra Models™ will provide a visual model of the problem. When students use Algebra Models™ they are able to see the difference between x and x^2 , y and y^2 , xy , and other variables and constants thus immediately resolving the confusion when combining terms.

NOTE: If confusion arises, it may be helpful to hold back the introduction of the 5-unit piece until students are comfortable with the idea of five units represented in one tile.

Support for Multiple Contexts

Algebra Models™ will support any context involving

the development of algebraic skills including general mathematics courses, traditional and reform pre-algebra and algebra I courses, and other secondary courses integrating algebra I topics. This support allows students to receive a more concrete idea of the algebra they are learning.

NCTM Recommendations

In the 2000 document, *Principals and Standards for Mathematics*, NCTM states that:

All students should learn algebra and furthermore instructional programs from pre-kindergarten through grade 12 should enable all students to:

- Understand patterns, relationships, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Analyze change in various contexts

In mathematics classrooms NCTM recommends that students:

- Work cooperatively
- Speak and write mathematically
- Persevere in their efforts to solve problems

And teachers should:

- Place a greater emphasis on student understanding and less on complex manipulations

Many teachers encounter difficulties when attempting to incorporate these recommendations into their algebra classrooms. Using Algebra Models™ offers students and teachers the opportunity to embody all of these recommendations into their algebra program

Timeline

When using Algebra Models™ to develop understanding in mathematics teachers should plan approximately one additional week to both introduce the manipulative and to allow students sufficient time to explore and practice prior to specific instruction of concepts. Although many teachers are concerned about valuable time they can be assured that the time spent introducing and exploring the tiles is critical to success and will allow them to advance further and with greater understanding. Once students are comfortable using these models real understanding will take place with less time necessary

for review and reteaching. Additionally, students become comfortable in their knowledge and more willing to take risks and explore more complex concepts. It is strongly recommended that teachers complete each problem prior to assigning it to students so that (s)he is aware of any unique situations that may arise.

Classroom Environment

According to research, effective long-term learning takes place when openness, discussion, exploration, and communication abounds and that the best learning environment is when students take an active role in the learning process rather than being a spectator in the educational game. Most experts will agree that classroom organization is critical to an open environment. Placing desks in groups rather than the traditional 5 or 6 rows neatly lined up will be more conducive to communication, exploration, and collaboration. There is a place in many classrooms when rote memorization is an effective method, as in learning the multiplication facts. However, for life-long understanding to take place, students need to explore, model, sketch, write, and discuss what is going on. Students are eager to take risks, share their findings and conclusions, and offer hypotheses when they have an opportunity to manipulate the situation. An open classroom environment and incorporating Algebra Models™ offers students such an opportunity.

When to Use This Activity Guide

Every effort has been taken to follow the development of the typical pre-algebra and algebra I course in this activity guide, however it is not intended to replace the classroom textbook. It has been designed as a supplement to be used in conjunction with any traditional or reform mathematics program that includes algebraic skills and concepts as well as any other algebra manipulative that uses an area model to explain key algebra topics. Our suggestion is to begin the school year with these materials, prior to the introduction of a textbook and then as an introduction to each new concept throughout the school year. By doing so, the frustration and confusion children typically encounter will be avoided. As students solve problems with Algebra Models™, preconceived negative ideas about algebra are gradually replaced with a more receptive and positive attitude. Algebra is difficult—many students encounter their first major academic obstacle in algebra yet they

are acutely aware of the significance algebra holds for them—high school graduation and acceptance into the college of their choice.

Layout

This manual is divided into four units. Each unit is sub-divided into specific topics. Each topic begins with teacher notes and examples for modeling each type of problem for the topic, followed by student lesson page(s). All units include one or more challenges; these may consist of explorations into other areas of mathematics or be extensions of the completed lessons. All student lesson pages are reproducible for classroom use.

Do's and Don'ts for Using Algebra Models™

Do expect students to model, sketch, write, and explain their thinking throughout the process of solving problems.

Do allow for some additional noise—if this becomes a detriment to the learning process for you or your students use a felt mat* to cover each desk while working with the models. This will reduce most noise created from plastic tiles being manipulated on students' desks.

**To make your own felt mats, inexpensive felt can be purchased at any fabric store.*

Don't expect to spend the same amount of time on each topic. In the beginning when students are learning how to use the manipulative some topics may take

longer to develop. However, as the year progresses, you and your students will be excited and surprised at how much more they accomplish due to a real understanding of concepts rather than rote memorization of algorithms that are often confusing and illogical to them.

Do take advantage of an exciting change in the attitudes and abilities of your students as they look forward to complex concepts. Most teachers who have used manipulatives in their classrooms are amazed at how in-depth mathematical concepts can be explored.

Do expect students to be active participants in the learning process as they model, sketch, write, and explain their thinking.

Topics to be covered include:

- Integers
- Absolute Value
- One- and Two-variable equations
- Addition and Subtraction
- Multiplication and Division
- Area and Perimeter
- Systems of Equations
- Factoring
- Solving Systems and Linear Equations in One or more Variables

Note: If the fraction and equations work mats (found on pages 125 and 126) are laminated, students will be able to use them for multiple activities. If laminated, students can use a SAFE-T® or other wet-erase marker to write directly on the mats.

Working with Fractions

Name _____ Date _____

Simplify each fraction.

1. $\frac{2}{4}$

2. $\frac{12}{16}$

3. $\frac{6xy}{2xy}$

4. $\frac{(5x - 10)}{(x - 2)}$

Find one equivalent fraction for each fraction.

5. $\frac{6}{3}$

6. $\frac{y}{(2 - y)}$

7. $\frac{x}{y}$

8. $\frac{xy}{2}$

Solve each proportion.

9. $\frac{4}{x} = \frac{12}{15}$

10. $\frac{3x}{12} = \frac{2}{4}$

Use Algebra Models™ to simplify fractions and to find equivalent fractions. Build each problem, then solve. Be sure to sketch your work and write each step throughout the process.

11. $\frac{3}{12}$

12. $\frac{7}{28}$

13. $\frac{12}{4}$

14. $\frac{8y}{4}$

15. $\frac{15x}{3x}$

16. $\frac{(3x - 6)}{(x - 2)}$

17. $\frac{3(2y - 1)}{(12y - 9)}$

Solve each proportion.

18. $\frac{5}{y} = \frac{1}{2}$

19. $\frac{2}{5} = \frac{6}{x}$

20. $\frac{2}{3} = \frac{8}{y}$

21. $\frac{x}{6} = \frac{2}{12}$